〈論　文〉

Optimal Inflation Tax in a Money-in-the Utility Model with Habit Formation

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Abstract

This paper introduces habit formation into a money-in-the utility model and investigates the optimal inflation tax. We show that the validity of the Friedman rule depends on the assumption of the satiation level for real money holdings. If the satiation level is infinite, the Friedman rule holds regardless of the existence of habit formation. On the other hand, when the satiation level is finite, the Friedman rule holds if parameters measuring the degree of decreasing current marginal utility are symmetric.

Keywords: Optimal Inflation tax; Habit formation; Friedman rule;
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1 Introduction

The central criterion of monetary policy is that the government should keep the nominal interest rate at zero. This criterion is called the Friedman rule for the optimal quantity of money, which insists that since the marginal cost of money supply is negligible, the marginal cost of money holdings is reduced to zero with the zero nominal interest rate. In the second best setting of the Ramsey problem, where the government cannot impose a lump-sum tax, however, the optimality of this rule is not so obvious. Adoption of the Friedman rule infers that the government imposes no inflation tax. Because the government needs financial resources, if the government applies the Friedman rule, the government must impose distortional taxes. Phelps (1973) concludes that

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the government should use both a distortional income tax and an inflation tax to finance spending. Kimbroug (1986), Correia and Teles (1996, 1999), and Chari et al. (1996, 1999) show, however, that despite the second best setting, in several monetary economies, the Friedman rule holds when household preferences are homothetic between consumption and money holdings and separable in leisure. Mulligan and Sala-i-Martin (1997) give a general discussion about the necessary conditions for preferences.[1]

These studies consider time-separable preferences and investigate the required conditions on preferences to maintain the optimality of the Friedman rule. Results on experimental observations, however, have provided little support for time-separable preferences. Then, we consider monetary models with habit formation, assuming typical time-dependent preferences, and investigate the conditions required on habit formation for the Friedman rule to hold.[2]

There are several antecedents that examine the Friedman rule when consumption forms habits. Our results may further explain why the Friedman rule is optimal in these models. Chugh (2007) discusses optimal monetary policy in a cash-credit model with habits that are formed according to consumption in previous periods. Chugh assumes that preferences are homothetic between credit and cash goods and concludes that the Friedman rule is always optimal. Faria (2001) considers a MIU model in which only consumption forms habits (i.e. real balance holdings do not form habits) and shows that the Freidman rule for optimal monetary growth holds regardless of habit parameters.

In this paper, we consider the conditions under which the Friedman rule holds in a MIU model where both consumption and money holdings form habits. We find that whether the Friedman rule holds or not depends on the assumption for the satiation level of money holdings. We claim in proposition 1 that when the satiation of money holdings is infinite, the Friedman rule always holds. A benevolent government should equalize the marginal utility of money holdings and the excess burden of inflation tax. Since the utility which households can gain from money holdings is satiated when holdings are

[1] Chari and Kehoe (1999) provide a survey of the literature of this field. After that, many studies investigate the robustness of Friedman rule in various models. For example, Adao et al. (2003) and Schmitt-Grohe (2004) study the optimality of the Friedman rule in imperfect competitive markets. Kocherlakota (2005) also provide a comprehensive survey on the Friedman rule.

[2] From several empirical observations, habit formation is often reported to significantly improve the explanation of consumer behavior. See for example, Naik and Moore (1996) and Fuhrer (2000).

infinite, marginal utility becomes zero. On the other hand, a natural assumption is that the marginal revenue of inflation tax must be zero because the government cannot obtain infinite revenue by increasing money supply. The tax revenue can be interpreted as the excess burden of tax for households. The zero revenue is identical to the zero excess burden of inflation tax and means a nominal interest rate of zero. Therefore, at the infinite satiation level of real money holdings, a zero nominal interest rate, that is the Friedman rule, is an optimal policy.

If money holdings are satiated at a finite level, however, the habit parameters have a crucial role. In proposition 2, we conclude that the Friedman rule holds as an interior solution only when the habit parameters for consumption and money holdings are identical. At a finite level of money supply, the revenue from the inflation tax does not hove to be zero. Then, the excess burden of inflation tax might not be zero at the satiation point. If household preferences are homothetic in consumption and money holdings, we can obtain a zero excess burden of inflation tax at the satiation point and the Friedman rule holds. To make preferences homothetic in consumption and money holdings, habit parameters are required to be symmetric between consumption and money holdings. If habit parameters are asymmetric, preferences are not homothetic and the Friedman rule does not hold. In proposition 3 we show the parameter conditions required for the Friedman rule to hold as a corner solution when habit parameters are asymmetric between consumption and money holdings.

This paper is organized as follows. In section 2, we provide the setup for a MIU model with habit formation and investigate conditions for the optimality of the Friedman rule. In section 3, we consider the optimal inflation tax. In section 4, we summarize the results.

2 The Money in the Utility Model

We consider a money-in-the utility (MIU) model with habit formation. Time is discrete and denoted by $t=0, 1, 2, \cdots$. There is an infinitely-lived representative household. Utility is a function of consumption, real balance holdings, and leisure. The preferences of the representative household are given by

$$\sum_{t=0}^{\infty}\beta^t U(\nu(X_t, M_t), l_t), \tag{1}$$

where β represents the discount factor, l_t is leisure, and $\nu(\cdot)$ is sub-utility. These preferences are weakly separable in leisure and have the usual assumptions of concavity and differentiability. X_t and M_t are, respectively, consumption, and real balance holdings in relative amounts, of instantaneous amount and habits, i.e. $X_t=\frac{x_t}{(h_{x,\,t-1})^{\eta_x}}$ and $M_t=\frac{m_t}{(h_{m,\,t-1})^{\eta_m}}$. These habits are formed by $h_{i,\,t+1}-h_{i,\,t}=\rho_i(i_t-h_{i,\,t})$, where $\rho_i\in[0,1]$, $i=x,m$.

The instantaneous budget constraint of the household is

$$(1-\tau_t)P_t(1-l_t)+N_t+(1+i_t)B_t\geq P_tx_t+N_{t+1}+B_{t+1}, \tag{2}$$

where τ_t, $(1-l_t)$, i_t, P_t and B_t represent, respectively, the income tax, labor supply, the nominal interest rate, price level, and bonds holdings from period t to $t+1$. The representative household faces budget constraint (2), the initial condition $N_{-1}=B_{-1}=0$, and the no Ponzi games condition. From these conditions, we gain the unique intertemporal budget constraint:

$$\sum_{t=0}^{\infty}I_tP_t(1-\tau_t)(1-l_t)\geq\sum_{t=0}^{\infty}I_tP_tx_t+\sum_{t=0}^{\infty}I_ti_tP_tm_t, \tag{3}$$

where $I_t\equiv1\Big/\prod_{s=0}^{t}(1+i_s)$ and $m_t=\frac{N_t}{P_t}$ represents real balances holdings.

For simplicity, we assume that one unit of labor produces one unit of consumption goods. Thus, the resource constraints which the economy faces in each period are given by

$$1-l_t\geq x_t+g_t, \tag{4}$$

where g_t is a given level of the government expenditures and is constant over time, i.e. $g_t\equiv g$.

3 The Optimal Money Supply

We consider the second best Ramsey problem. The representative household maximizes utility (1) subject to the intertemporal budget constraint (3).

The first order conditions are

$$\beta^t[U_{x_t}+V_{x_t}]=\lambda I_tP_t,$$

$$\beta^t[U_{m_t}+V_{m_t}]=\lambda I_ti_tP_t,$$

$$\beta^tU_{l_t}=\lambda I_tP_t(1-\tau),$$

where $U_{x_t}\equiv U_{X_t}\cdot(h_{x,t-1})^{-\eta_x}$, $U_{m_t}\equiv U_{M_t}\cdot(h_{m,t-1})^{-\eta_m}$, $V_{x_t}\equiv\sum_{T=t+1}^{\infty}\beta^{T-t}U_{X_T}\frac{\partial X_T}{\partial x_t}$, $V_{m_t}\equiv\sum_{T=t+1}^{\infty}\beta^{T-t}U_{M_t}\frac{\partial M_T}{\partial m_t}$ and λ is the Lagrange multiplier.

From these three equations, we derive the following conditions:

$$(U_{x_t}+V_{x_t})\cdot i_t=(U_{m_t}+V_{m_t}), \tag{5}$$

$$(U_{x_t}+V_{x_t})\cdot(1-\tau_t)=U_{l_t}, \tag{6}$$

$$U_{x_t}+V_{x_t}=\frac{\beta(1+i_{t+1})P_t}{P_{t+1}}(U_{x_{t+1}}+V_{x_{t+1}}). \tag{7}$$

In the second best Ramsey problem a benevolent government chooses a sequence $\{x_t, m_t, l_t\}_{t=0}^{\infty}$ to maximize (1) subject to the resource constraint (4) and the implementability constraint. By substituting (5), (6), and (7) into (3), the implementability constraint is derived as

$$\sum_{t=0}^{\infty}\beta^t[(U_{x_t}+V_{x_t})x_t+(U_{m_t}+V_{m_t})m_t-U_{l_t}(1-l_t)]\leq 0. \tag{8}$$

The first order conditions of the government problem are

$$(U_{x_t}+V_{x_t})-\mu[(U_{x_t}+V_{x_t})+(U_{x_tx_t}+V_{x_tx_t})x_t+(U_{x_tm_t}+V_{x_tm_t})m_t]-\Psi_t=0, \tag{9}$$

$$(U_{m_t}+V_{m_t})-\mu[(U_{m_t}+V_{m_t})+(U_{x_tm_x}+V_{x_tm_t})x_t+(U_{m_tm_t}+V_{m_tm_t})m_t]=0, \tag{10}$$

$$U_{l_t}-\mu[U_{l_t}-U_{l_tl_t}(1-l_t)]-\Psi_t=0, \tag{11}$$

where $\Psi_t\equiv\frac{\psi_t}{\beta^t}$, and μ and ψ_t are the multipliers of the implementability constraint and resource constraints in each period. To concentrate our discussion on the steady state, we omit the time subscript t. From (5), the Friedman rule, $i=0$, implies $(U_m+V_m)=0$. Therefore, from (10), when the Friedman rule is optimal as an interior solution, if the following equation is satisfied:

$$(U_{xm}+V_{xm})x+(U_{mm}+V_{mm})m=0. \tag{12}$$

The establishment of (12) depends on the satiation level of real balance holdings in the sub-utility function $\nu(\cdot)$. Here, we introduce an assumption regarding the satiation level.

Assumption 1. The satiation level of real balances in relative amounts is infinite, i.e. $\lim_{M\to\infty}\left(\frac{\partial\nu}{\partial M}\right)=0$. If the government increases the rate of printing new money to infinity, the marginal revenue of inflation tax $i\cdot m$ must converge to zero.

Correia and Teles (1999) show that the conditions required for the validity of the Friedman rule in the MIU model when habit formation is absent. The second assumption in Assumption 1 guarantees that the additional marginal revenue from inflation tax becomes zero when the government provides sufficient large real balance holdings and the nominal interest rate is zero. Correia and Teles (1999) point out that this assumption is natural in monetary economies.

Proposition 1

When household preferences satisfy Assumption 1, the Friedman rule is optimal regardless of habit parameters.

Proof

From (5), the Friedman rule $i=0$ is equivalent to $(U_{m_t}+V_{m_t})=0$. Then we have $U_m+V_m=U_M m^{-\eta_m}\left(1-\dfrac{\beta\eta_m\rho_m}{1-\beta(1-\rho_m)}\right)$ in stationary state. At stationary state, from the Assumption 1, we have

$$\lim_{m\to\infty}(U_m+V_m)=0. \tag{13}$$

We should assume that, when the government increases m_t to infinity, the marginal revenue of inflation tax $i\cdot m$ must converge to zero. That is $\lim_{m\to\infty}\dfrac{\partial((U_m+V_m)m)}{\partial m}=\lim_{m\to\infty}((U_{mm}+V_{mm})m+(U_m+V_m))=0$. From (13) and this equation, $\lim_{m\to\infty}(U_{mm}+V_{mm})m=0$ must hold. This is the second term on the left hand side in (12). Next, the first terms on the left hand side of (12) are $U_{x_t m_t}=\dfrac{U_{X_t M_t}}{(h_{x,\,t-1})^{\eta_x}(h_{m,\,t-1})^{\eta_m}}$ and $V_{x_t m_t}=\sum_{T=t+1}^{\infty}\beta^{T-t}U_{X_T M_T}\dfrac{\partial x_T}{\partial h_{x,\,T-1}}\dfrac{\partial h_{x,\,T-1}}{\partial x_t}\dfrac{\partial m_T}{\partial h_{m,\,T-1}}\dfrac{\partial h_{m,\,T-1}}{\partial m_t}=\beta\eta_x\eta_m\rho_x\rho_m\sum_{T=t}^{\infty}\beta^{T-t}U_{X_{T+1}M_{T+1}}\dfrac{x_{T+1}m_{T+1}}{(h_{x,\,T})^{\eta_x+1}(h_{m,\,T})^{\eta_m+1}}(1-\rho_x)^{T-t}(1-\rho_m)^{T-t}$. Then in the stationary state, we have $(U_{xm}+V_{xm})x=A\cdot U_{XM}x^{-\eta_x}m^{-\eta_m}$ where $A\equiv 1+\dfrac{\beta\eta_x\eta_m\rho_x\rho_m}{1-\beta(1-\rho_x)(1-\rho_m)}$. Because $\lim_{m\to\infty}m^{-\eta_m}=0$, we have $\lim_{m\to\infty}((U_{xm}+V_{xm})x)=0$. Therefore, since $i=0$ leads to (12), the Friedman rule is optimal. Q. E. D.

Faria (2001) sets up a MIU model in which only consumption forms habits, and shows that, even if consumption forms habits, the super-neutrality of monetary growth holds and the Friedman rule for optimal monetary growth is optimal. Similar to Faria's result, proposition 1 says that, even when consumption is habit forming, the Friedman rule for the optimal money supply is the solution to the second best Ramsey problem under Assumption 1. Moreover, even if real balance holdings form habits, the Friedman rule is always optimal.

At the optimal point, the marginal utility of real balances equals the marginal excess burden of inflation tax. At the satiation level, a marginal utility of zero is achieved through infinite real money holdings regardless of habit formation. Since money is a free good, the marginal excess burden does not include the shadow values of the resource constraints. On the other hand, Assumption 1 implies that the marginal revenue of inflation tax converges to zero if the government supplies an infinite quantity of money. Then the marginal excess burden also becomes zero and (10) is satisfied when $m=\infty$.

The result in proposition 1 is derived from Assumption 1 directly.

Do habit parameters have a relationship with the Friedman rule in all MIU models? The answer is no. It is well known that other monetary models (i.e. cash-credit models and shopping time models) are equivalent to some MIU models. In particular, Correia and Teles (1999) argue that a cash-credit good model is equivalent to a MIU model with a finite satiation level for real balance holdings and show that the Freidman rule is optimal under the following assumption.

Assumption 2. The satiation level is a linear function of consumption in relative quantities, i.e. $U_M>0$ if $M<M^*(X)$, $U_M\leq 0$ if $M^*(X)\leq M$, and $M^*=kX$, where M^* is the satiation of real balances in relative quantities, and k is a positive constant.

Assumption 2 infers a unitary elasticity of money demand with respect to consumption. The following proposition sheds light on the optimality of the Friedman rule under Assumption 2 instead of Assumption 1.

Proposition 2

Suppose that the satiation level of real balance holdings satisfies Assumption 2. If $\eta_x=\eta_m$, the Friedman rule is optimal.

Proof

In the stationary state, from $x=h_x$ and $m=h_m$, $X=x^{1-\eta_x}$ and $M=m^{1-\eta_m}$. The condition of the case 2 is rewritten as $m^{1-\eta_m}=kx^{1-\eta_x}$. If $\eta\equiv\eta_x=\eta_m$, this equation is

$$m^*=k^{\frac{1}{1-\eta}}x, \tag{14}$$

where m^* is the satiation level of real balance holdings. At the satiation level, from $U_m+V_m=0$, we have $-\dfrac{dm^*}{dx}=\dfrac{U_{xm}+V_{xm}}{U_{mm}+V_{mm}}$. By this equation, (12) is rewritten as

$$m^*(U_{mm}+V_{mm})\left(1-\frac{dm^*}{dx}\,\frac{x}{m^*}\right)=0. \tag{15}$$

From (14), we have that $(dm^*/dx)(x/m^*)=1$, and so the second term in parenthesis in (15) is zero. Q. E. D.

Chari et al. (1996) prove that the optimality of the Friedman rule is derived from homothetic and separable preferences. Their result is explained using the basic optimal taxation criterion advocated by Atkinson and Stiglitz (1972). They conclude that uniform taxation is the optimal policy when the indifference map is homothetic. In our model, a distortional tax is only placed on labor income. Proposition 2 claims that symmetry in habit parameters is necessity for the Friedman rule to hold. If $\eta_x = \eta_m$, preferences are homothetic in x and m. Then, from the basic optimal taxation rule of Atkinson and Stiglitz, the government should not impose a positive tax rate on m. The nominal interest rate is a tax rate on money holdings, and the government should set the nominal interest rate at zero. Therefore, the Friedman rule, $i=0$, is optimal as an interior solution.

Even when $\eta_x \neq \eta_m$, it is possible that $i=0$ is optimal as a corner solution because of the non-negativity constraint on the nominal interest rate.[3] We consider whether there is a glut in real balances or not.

Proposition 3

Suppose that the satiation point of real balance holdings satisfies Assumption 2. If $\eta_x < \eta_m$, the Friedman rule is not the optimal policy. If $\eta_x > \eta_m$, the Friedman rule is the optimal policy as a corner solution.

Proof

Since $m^{1-\eta_m} = kx^{1-\eta_x}$, we have $(1-\eta_m)m^{-\eta_m}dm = k(1-\eta_x)x^{-\eta_x}dx$ or $\frac{dm}{dx} = \frac{1-\eta_x}{1-\eta_m}\frac{m}{x}$. Defining L as the Lagragian of the government problem, we reconsider the first-order derivative in m and estimate at $i=0$, which implies $(U_m + V_m) = 0$. Then we have

$$\begin{aligned}\left.\frac{\partial L}{\partial m}\right|_{i=0} &= -\mu[(U_{xm}+V_{xm})x + (U_{mm}+V_{mm})m] \\ &= -\mu m(U_{mm}+V_{mm})\left(1-\frac{dm}{dx}\frac{x}{m}\right) \\ &= -\mu m(U_{mm}+V_{mm})\left(1-\frac{1-\eta_x}{1-\eta_m}\right), \qquad (16)\end{aligned}$$

where $U_{mm} + V_{mm} < 0$. First, we consider $\eta_x < \eta_m$, where the right hand side of (16) is negative. The optimal level of real balances is less than one at $i=0$. Hence, the optimal nominal interest rate is $i>0$. On the other hand, if $\eta_x > \eta_m$, the right hand side of (16) is positive. However, when $(U_m + V_m) = 0$, the satiation level of the household is attained, so the real balance holdings of households do not increase, even if the govern-

[3] When government can use consumption taxes, Friedman's rule may be optimal without homothetic preference but the optimal tax rates will not be determined.

ment increases the rate of printing new money. Thus, in this case, the Friedman rule holds as a corner solution. Q. E. D.

Suppose $\eta_x = \eta_m$ as a base point. If η_m increases, then the marginal utility of real balance holdings decreases. Hence, the government must reduce money supply, and the nominal interest rate increases. Therefore, the Friedman rule, $i=0$, is not the optimal policy. On the other hand, if η_x increases, the marginal utility of good x decreases compared to the marginal utility of real balances. In this model, since a distortional tax is placed on labor income and household preferences are separable in leisure, the government cannot control the marginal utility of good x through a distortional labor income tax. Then the government tries to equalize marginal utilities by printing new money. But, since the nominal interest rate is bounded by a non-negativity constraint, the nominal interest rate does not decrease when $i=0$. Therefore, in this case, the Friedman rule is optimal as a corner solution.

4 Concluding Remarks

We investigated the effects of habit formation on the optimality of the Friedman rule. In a standard MIU model with habit formation, whether the Friedman rule holds or not depends on the assumptions associated with the satiation level of real money holdings. If, as in Assumption 1, the satiation level of money holdings is infinite, the Friedman rule holds regardless of habit parameters. This result stems from the assumption that the excess burden of inflation tax must be zero when money supply is infinite. This assumption guarantees that the first order condition of optimal money supply for a benevolent government always holds at the satiation level of money holdings. Homotheticity is no longer required for the Freidman rule to hold. The result of proposition 1 is derived from the property that money is not an intermediate good but a final good in the MIU model.

On the other hand, if the satiation point of money holdings is finite, the Friedman rule depends critically on the habit parameters. Under Assumption 2 (finite satiation of money holdings), the Friedman rule holds when preferences are homothetic. If the satiation point of money holdings is finite, the excess burden of inflation tax must not be zero at the satiation point. Then, homothetic preferences are required to obtain the zero excess burden of inflation tax at the satiation point. In our model, symmetric habit parameters provide homothetic preferences between consumption and money holdings. Proposition 2 claims that under Assumption 2, the Friedman rule holds as an interior solution only when habit parameters are identical, i.e $\eta_x = \eta_m$.

We note that the Friedman rule might hold as a corner solution of the second best Ramsey problem. Proposition 3 requires that the government sets a nominal interest rate of zero when the habit parameter for consumption dominates the parameter for money holdings, i.e $\eta_x > \eta_m$. Because the higher habit parameter decreases marginal utility, the government should increase the money supply over the satiation level. However, households are not willing to hold more money at the satiation point, and the government must set the nominal interest rate to zero.

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要　　約

この論文では，家計の選好に，過去の消費と貨幣保有からの習慣形成（habit）が存在する場合の，最適な貨幣供給量を分析している。最適貨幣発行ルールの一つにフリードマンルールがある。これによると，一括税が使用可能な場合，政府は名目利子率を0にするように貨幣を発行すべきである。この分析を，一括税がない動学モデルに拡張した研究として，Chari et al.（1996），およびCorreia and Teles（1999）などがある。これらの研究では，効用関数が相似形（homothetic）であれば，フリードマンルールが成立することを示した。これら従来の研究では，各期間の効用関数が独立である（時間独立型の効用関数）と仮定されている。しかしながら，最近の実証研究であるNaik and Moore（1996），Fuhrer（2000）などにより，各期間の効用関数が相互依存をしている可能性が指摘されてきた。そこで本稿では，これらの研究に時間依存型の効用関数を導入することで拡張を試みている。

本稿では，時間依存型の効用関数の代表的なモデルである習慣形成（habit formation）をMoney-in-the-Utilityモデルに導入した場合，フリードマンルール成立条件がどのように変化するのかを分析している。本稿の結論として，フリードマンルールの成立は，モデルにおける貨幣保有の飽和点（satiation level of money holdings）に大きく依存する。

命題1では，飽和点が無限大の場合，フリードマンルールは習慣形成（habit）のパラメーターの値に依存せず，常に成立することを示した。この結論は，貨幣を無限大に発行した場合，貨幣発行からの政府収入（marginal revenue）はゼロになるという仮定から得られる。一方，命題2では，貨幣保有の飽和点が有限の場合，消費財と貨幣保有の習慣形成のパラメーターが一致する場合にのみ，フリードマンルールは成立することを示した。しかし，パラメーターがそれ以外の値をとった場合，一般的には，フリードマンルールは成立しない。これは，二つのパラメーターが一致する場合に，選好が習慣形成による効果を含めても相似形となり，従来の議論と同様の結論が得られるためである。命題3では，二つの習慣形成のパラメーターが異なる場合でも，フリードマンルールが端点解（corner solution）として成立する可能性を示している。

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